

Is it worth guessing the last 10 questions on the UKMT Challenge?

There are 25 questions in a Challenge Paper. Questions 1-15 are each worth 5 marks. Questions 16-20 are each worth 6 marks but if you get the question wrong you lose a mark. Questions 21-25 are each worth 6 marks but if you get a question wrong you lose 2 marks.

Is it worth guessing the last 10 questions on the UKMT to try to increase your total mark?

Possible Solutions

Expected Frequency

Q16-20

Let X be the number of possible marks on a question. Then for Q16:

x	-1	6
P(X=x)	0.8	0.2

So $E(X) = -1 \times 0.8 + 6 \times 0.2 = 0.4$ marks

Hence, you can expect to get 0.4 marks per question on Q16 if you guess.

For all five questions from Q16-20, **$E(X) = 0.4 \times 5 = 2$ marks**

Q21-25

Let X be the number of possible marks on a question. Then for Q21:

x	-2	6
P(X=x)	0.8	0.2

So $E(X) = -2 \times 0.8 + 6 \times 0.2 = -0.4$ marks

Hence, you can expect to lose 0.4 marks per question on Q21 if you guess.

For all five questions from Q21-25, **$E(X) = -0.4 \times 5 = -2$ marks**

Therefore if you guess all of the last 10 questions, you can expect to get an extra $2 + -2 = 0$ marks.

Conclusion: Guess Q16-20 and you can expect to be awarded an extra 2 marks. Do not guess Q21-25.

Binomial Distribution

Q16-20

If you get 1 right and get the other 4 wrong then you would get 6 marks + (4 x -1 marks) = 2 marks.

What's the probability of getting one or more right?

Let X be the number of guesses you get right.

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X \geq 1) = 1 - \binom{5}{0} (0.2)^0 (0.8)^5$$

$$P(X \geq 1) = 0.672$$

So the probability of getting one or more right and getting extra marks is 67.2%.

Q21-25

If you get 1 right and get the other 4 wrong then you would get 6 marks + (4 x -2 marks) = -2 marks.

If you get 2 right and get the other 3 wrong then you would get 12 marks + (3 x -2 marks) = 6 marks.

Hence you would need to get 2 or more right for it to be beneficial to guess.

$$P(X \geq 2) = 1 - P(X \leq 1)$$

$$P(X \geq 2) = 1 - 0.73728$$

$$P(X \geq 2) = 0.263$$

So the probability of getting 2 or more right and getting extra marks is 26.3%.

Conclusion: Guess Q16-20 and leave Q21-25 blank.