Differences

TASK 1

With any sequence, you can find the differences between each term. Some sequences have a **first difference** which is **constant**. For example, the sequence below has a first difference which is always four.

\[
6, \quad 10, \quad 14, \quad 18, \quad 22, \ldots
\]

First Difference: \[4 \quad 4 \quad 4 \quad 4\]

Here is a sequence in which the **third difference** is **constant**. The third difference is always 6.

\[
2, \quad 9, \quad 28, \quad 65, \quad 126, \quad 217, \ldots
\]

First Difference: \[7 \quad 19 \quad 37 \quad 61 \quad 91\]

Second Difference: \[12 \quad 18 \quad 24 \quad 30\]

Third Difference: \[6 \quad 6 \quad 6\]

Question 1: Can you find a sequence in which the second difference is constant? Is there a strategy to do this with ease?
We can represent sequences in different forms.

<table>
<thead>
<tr>
<th>First Difference:</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table Form:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( y )</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>(1,5)</td>
<td>(2,8)</td>
<td>(3,11)</td>
<td>(4,14)</td>
<td>(5,17)</td>
</tr>
</tbody>
</table>

Graphical Form:

Question 2: Complete each section below to find the equation of the line.

<table>
<thead>
<tr>
<th>First Difference:</th>
<th>3, 7, 11, 15, 19,...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table Form:</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td></td>
</tr>
<tr>
<td>(1,5)</td>
<td></td>
</tr>
</tbody>
</table>

Graphical Form:

\( y = 3x + 2 \)
Question 3: Here is another sequence with a first difference which is constant.

8, 13, 18, 23, 28, ....

Could you find the equation of the straight line without completing a table or drawing a graph? How might you find the gradient and y-intercept? Explain your reasoning.

Use your equation to find the 50\textsuperscript{th} term of the sequence above?

What do the variables represent in the context of a sequence?

The variable x represents ........................................................................................................

The variable y represents ........................................................................................................

Question 4: Find the equation for this sequence in a table:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
<td>-6</td>
</tr>
</tbody>
</table>
Question 5: Find the equation for this sequence. Notice that the variables have now changed in symbolism, and that the table is now vertical.

<table>
<thead>
<tr>
<th>n</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

Question 6: Draw a vertical table for the following equation: \( C = 2t + 30 \)
Question 7: Can you make an equation for the even number sequence and an equation for the odd number sequence?

Question 8: Describe what you learnt from this task.

1. …………………………………………………………………………………………………………………
   …………………………………………………………………………………………………………………

2. …………………………………………………………………………………………………………………
   …………………………………………………………………………………………………………………
TASK 2

You have just completed a task based on sequences with a constant first difference. **When a sequence has a constant first difference, it is a straight line on a co-ordinate grid.** Below I have given equations which are not straight lines.

1. Generate the first 5 terms of each sequence from its equation.
2. Calculate the first, second and third difference, if needed, for each sequence.
3. Graph the equations in Geogebra.
4. Sketch each graph in the boxes provided.

<table>
<thead>
<tr>
<th>Quadratic Equation</th>
<th>Quadratic Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 + 1 )</td>
<td>( y = 2x^2 + 1 )</td>
</tr>
</tbody>
</table>
Cubic Equation
\[ y = x^3 + 1 \]

Cubic Equation
\[ y = x^3 - 3 \]

Question 5: Describe anything you notice about the connection to the equation, the sequence and the graphs.

Quadratic equations:
Cubic equations:

Question 6: For any point you pick on a straight line graph, the gradient is constant. Is this also the case for a quadratic or cubic graph? Can you just as easily determine the gradient by looking at the equation? Explain your answer.
Question 7: Use your knowledge to find the equation of the sequence below.

5, 11, 21, 35, 53, ...

Question 8: What would this sequence look like on a graph? Draw an approximate sketch.
Question 9: Make up a **cubic sequence** of your own.

**Task 3**

Below I have provided some more interesting sequences. Find the first, second, third and fourth difference in each case. Do you notice any patterns within these differences? Can you find an equation to represent these sequences? If not, explain why you think you cannot.

Fibonacci Sequence:

\[0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\]
Prime Number Sequence

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ….

Exponential Sequence

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, ….

Challenges:

* Represent the exponential sequence above with visually. What is the 20th term be? Looking at the differences, is there anything you can say about exponential growth in comparison to linear growth or quadratic growth?

* Justify or prove why the second difference is constant for a quadratic sequence.

** Do the same for a cubic sequence.